Answers

Answers to further practice questions

1.1 $\frac{1}{10}$ = 10% $\frac{3}{10}$ = 30% $\frac{7}{10}$ = 70% $\frac{9}{10}$ = 90%

$\frac{1}{5}$ = 20% $\frac{2}{5}$ = 40% $\frac{3}{5}$ = 60% $\frac{4}{5}$ = 80%

$\frac{1}{8}$ = 12.5% $\frac{3}{8}$ = 37.5% $\frac{5}{8}$ = 62.5% $\frac{7}{8}$ = 87.5%

$\frac{1}{20}$ = 5% $\frac{3}{20}$ = 15% $\frac{7}{20}$ = 35% $\frac{9}{20}$ = 45%.

1.2 Multiplying by 4, '21 out of 25' is equivalent to '84 out of 100', or 84%. Multiplying by 5, '17 out of 20' is equivalent to '85 out of 100', or 85%. So, the second mark is the higher proportion of the total marks for the test.

1.3 For the first school, dividing by 6, '126 out of 600' is equivalent to '21 per hundred', or 21%. For the second school, '104 pupils out of 400' is '52 pupils per 200' or '26 pupils per 100', i.e. 26%. The second school has the larger proportion of pupils with English as an additional language.

2.1 For spelling, $\frac{19}{25} \rightarrow \frac{38}{50} \rightarrow \frac{76}{100}$ = 76%.

For mathematics, $\frac{42}{70} \rightarrow \frac{6}{10} \rightarrow$ 60%.

2.2 Unsatisfactory: $\frac{35}{250} \rightarrow \frac{7}{50} \rightarrow \frac{14}{100}$ = 14%.

Good: $\frac{130}{250} \rightarrow \frac{13}{25} \rightarrow \frac{26}{50} \rightarrow \frac{52}{100}$ = 52%.

Satisfactory: 100% – (14% + 52%) = 34%.

2.3 School R, 80%; School Q, 74%; School P, 60%.

3.1 The sequence should be 1.7, 1.8, 1.9, 2.0 (or 2), … the pupil is incorrectly thinking that after 'one point nine' comes 'one point ten'. The number 1.10 is the same as 1.1, which does not come after 1.9.

3.2 Tuesday and Wednesday have proportions greater than 0.08.

3.3 The target is less than 8%; Monday, 7.5%; Tuesday, 10%, Wednesday, 9%; Thursday, 7.9%; Friday, 0.9%.

4.1 a) True. b) False. c) True. d) False. e) False.

5.1 a) This set of data could potentially contain a large number of different values, such as 5.0, 5.1, 5.2, 5.3, and so on; so it will probably have to be grouped into intervals, such as 5.0–5.2, 5.3–5.5, 5.6–5.8, and so on.

b) This variable will probably take only whole-number values from 0 to 10, so grouping will be unnecessary.

c) Point scores may take a large number of different values. An able pupil with nearly all A* and A grades in eight subjects might score around 60 points, for example. It will be necessary to group this data into intervals.

5.2 a) About 25 b) About 60 c) About 2000 classes.

d) No. The number of classes in this range is less than 1300, that is, less than 65% of the total.

6.1 b) and d) are continuous variables.

6.2 b) The stop-watch used to time the pupils will measure their times only to the nearest something. So, even though time-taken is theoretically a continuous variable, it might be rounded, for example, to the nearest tenth of a second. The times might range from about 14.5 up to maybe 24 seconds, so the rounded times could be grouped conveniently in intervals such as 14.5–15.4, 15.5–16.4, 16.5–17.4, and so on, giving about 10 groups for display in a bar chart. This is, of course, just one possible suggestion for handling the data.

d) A very rotund teacher might have, say, a girth of 110 cm and a height of 145 cm, giving a ratio of $\frac{110}{145}$ = 0.75862068966 on a calculator. By contrast, a super-model teacher might have a girth of only 50 cm but a height of 190 cm, giving a ratio of $\frac{50}{190}$ = 0.26315789474. It would be sufficient to round these ratios to two decimal places, suggesting a range of values from about 0.26 to 0.76. One way of grouping this rounded data then would be in intervals such as 0.25–0.29, 0.30–0.34, 0.35–0.39, and so on, giving possibly about 10 groups for display on a bar chart.

7.1 I would reply, 'Why don't you just count them?' However, if you are asked this daft question in a numeracy test the correct answer is 12.

7.2 32.

7.3 The calculator result is 113.74825, so the VAT payable is £113.75.

8.1 $\frac{1}{10} = 0.1$ $\frac{3}{10} = 0.3$ $\frac{7}{10} = 0.7$ $\frac{9}{10} = 0.9$

$\frac{1}{5} = 0.2$ $\frac{2}{5} = 0.4$ $\frac{3}{5} = 0.6$ $\frac{4}{5} = 0.8$

$\frac{1}{8} = 0.125$ $\frac{3}{8} = 0.375$ $\frac{5}{8} = 0.625$ $\frac{7}{8} = 0.875$

$\frac{1}{20} = 0.05$ $\frac{3}{20} = 0.15$ $\frac{7}{20} = 0.35$ $\frac{9}{20} = 0.45$.

8.2 a) $0.175 = \frac{175}{1000} = \frac{7}{40}$ b) $0.007 = \frac{7}{1000}$.

8.3 Using a calculator, $\frac{4}{7} = 0.571$ (approximately), $\frac{879}{1500} = 0.586$. The second of these is the larger.

9.1 $5\% = \frac{1}{20}$ $10\% = \frac{1}{10}$ $18\% = \frac{9}{50}$ $30\% = \frac{3}{10}$ $37\% = \frac{37}{100}$.

9.2 17.5% = '17.5 in 100' = '35 in 200' = $\frac{35}{200} = \frac{7}{40}$ (cancelling 5). So VAT is applied at the rate of £7 in every £40.

9.3 12.5% is half of 25%, i.e. half of a quarter = one eighth ($\frac{1}{8}$). 87.5% must therefore be seven-eighths ($\frac{7}{8}$). The sum of the two fractions must be 1. It's worth memorising these equivalents, as well as $\frac{3}{8} = 37.5\%$ and $\frac{5}{8} = 62.5\%$.

10.1 a) 48% of £75 gives the same result as 75% of £48, i.e. $\frac{3}{4}$ of £48 = £36.

b) 35% of £60 gives the same result as 60% of £35, i.e. $\frac{3}{5}$ of £35 = £21.

10.2 895 ÷ 14.5.

10.3 a) False: '28 sets of zero' gives zero, so 28   0 = 0.

b) True: note that 28   0 = 0   28.

c) False: division by zero is meaningless.

d) True: 0 ÷ 28 could mean 'how many sets of 28 pupils are needed to make zero pupils in total?' Answer: zero sets!

11.1 30 – (18 – 10) = 30 – 8 = 22, but (30 – 18) – 10 = 12 – 10 = 2.

Because these results are different, subtraction is not associative. In general, A – (B – C) is not equal to (A – B) – C (unless C = 0). The situation described corresponds to 30 – (18 – 10).

11.2 160 ÷ (8 ÷ 4) = 160 ÷ 2 = 80, but (160 ÷ 8) ÷ 4 = 20 ÷ 4 = 5.

Because these results are different, division is not associative. In general, A ÷ (B ÷ C) is not equal to (A ÷ B) ÷ C (unless C = 1 or –1).

11.3 28   25 = (7   4)   25 = 7   (4   25) = 7   100 = 700. Cost = £700.

12.1 (a) (100 + 80)   8 = (100   8) + (80   8) = 800 + 640 = 1440.

(b) (200 – 20)   8 = (200   8) – (20   8) = 1600 – 160 = 1440.

Cost = £1440.

12.2 a) First find the total cost of one each of the two books, £12 + £4 = £16, then multiply this by the number of pupils: 25   £16 = £400.

b) First find the total cost of textbooks (25   £12 = £300) and the total cost of workbooks (25   £4 = £100), then add these: £300 + £100 = £400.

12.3 a) (£700 + £630) ÷ 7 = (£700 ÷ 7) + (£630 ÷ 7) = £100 + £90 = £190.

b) (£1400 – £70) ÷ 7 = (£1400 ÷ 7) – (£70 ÷ 7) = £200 – £10 = £190.

13.1 a) Doing the operations in the order entered, the four-function calculator would give the result as 3.

b) Giving precedence to the division, the scientific calculator would give the result as 9.

13.2 The operations have been done in the order entered. This is using a basic four-function calculator. The result displayed is correct in this context.

14.1 My estimate was around £84 (£20 + £9 + £15 + £36 + £4).

Actual cost = £85.75.

14.2 C looks most likely. We need less than £2 for each of less than 70 pupils, so the cost should be under £140. Actual cost is £131.92.

14.3 The calculator result of 181.7 has been misinterpreted as £181.07. It should be £181.70.

15.1 $\frac{38}{190} = \frac{2}{10}$ = 20% (no calculator required)

$\frac{23}{190}$ = 0.12105263158 = 12.1% to one decimal place

$\frac{19}{190} = \frac{1}{10}$ = 10% (no calculator required)

$\frac{5}{190}$ = 0.02631578947 = 2.6% to one decimal place

$\frac{1}{190}$ = 0.00526315789 = 0.5% to one decimal place.

16.1 Females: C, 19.5%; D, 18.3%, E, 14.2%; N, 8.2%.
Males: C, 18.5%, D, 17.9%, E. 14.0%; N, 8.5%.

16.2 Giving these to the nearest whole percent would not discriminate sufficiently between the data. For example, both females and males would have 18% for grade D. It would also exaggerate some differences, e.g. giving one whole percent difference between females and males in the N-grade category (using 8% and 9% respectively, instead of 8.2% and 8.5%).

17.1 To order these populations, look first at the power of 10 and then at the digits. The order is: UK, Japan, India, China.

17.2 £290 000 000 or £2.9 $\quad 10^8$.

17.3 4.4820717 $\quad 10^{-3}$ = 0.0044820717 = approximately 0.4%.

18.1 One method is to round the £4.95 up to £5. Then 24   £5 = £120. Subtract 24   5p = £1.20. Answer £118.80.

18.2 One method is to use factors, writing 48 as 4   12. Then 125   4 = 500 and 500 x 12 = 6000. Answer £6000.

18.3 The 97 is close to 100, so think of it as 100 – 2 – 1. Then we need (240   100) – (240   2) – (240   1) = 24 000 – 480 – 240 = 23 520 – 240 = 23 280.

18.4 45   74 = 45   2   37 = 90   37 = 100   37 – 10   37 = 3700 – 370 = 3330
Area = 3330 square metres.

44   75 = 11 x 2   2   75 = 11   2   150 = 11   300 = 3300
Area = 3300 square metres, which is smaller.

19.1 You could start by dividing both numbers by 3 to give 48 ÷ 3 = 16. Or you could think of the 144 as 180 – 36, which when divided by 9 gives 20 – 4 = 16. Or you could break the 144 up into 90 + 54, giving 10 + 6 = 16.

19.2 6035 ÷ 85 = 12070 ÷ 170 (doubling both numbers) = 1207 ÷ 17 (dividing by 10). To divide 1207 by 17, I would start with 1700 (100 17), which is 493 too much. The 493 can be split up into 340 + 153 = 340 + 170 – 17, each bit of which can be divided easily by 17. Answer: 100 – (20 + 10 – 1) = 71, i.e. £71 per pupil.

19.3 For 893 ÷ 24, I would start with 30 24 = 720. So I need another 173. Next, 5 24 = 120, so I need another 53. Then, 2 x 24 = 48, which leaves me just 5 short. The result is 30 + 5 + 2 = 37, with 5 remainder. This 5 is less than half a mark per pupil, so to the nearest whole number the average mark is 37.

20.1 a) 12.5% of 160 = $\frac{1}{8}$ of 160 = 20; b) 30% of 220 = $\frac{3}{10}$ of 220 = 66.

20.2 50% = 120; 25% = 60; 1% = 2.4; 2% = 4.8. Adding these, 78% = 187.2. So about 187 pupils, or 188 to pass the target.

20.3 The percentage not reaching level 4 is 100% – 40% – 36% = 24%. So we need 24% of 125. 20% = $\frac{1}{5}$ which is 25; 4% = $\frac{1}{25}$ which is 5. Total = 30.

21.1 Key sequence for first method: 279 47.8 ÷ 100 =
Key sequence for second method: 279 47.8 % (procedure not universal)
Key sequence for third method: 0.478 279 =
The result is 133.362, so to surpass this percentage 134 pupils must achieve the required grades.

22.1 Stepping from 14.87 to 14.9 to 15 to 20 and then to 100, the difference between 14.87% and 100% is 0.03 + 0.1 + 5 + 80 = 85.13%.

22.2 2.970 + 34.000 + 1.085 = 38.055 m.

22.3 3.620 – 2.085 = 1.535 m.

23.1 132.

23.2 330.

24.1 The areas method gives four multiplications: 20 30, 20 9, 8 30, 8 9, giving 600 + 180 + 240 + 72 = 1092 hours.

24.2 The areas method gives six multiplications: 100 70, 100 2, 40 70, 40 2, 2 70, 2 2, giving 7000 + 200 + 2800 + 80 + 140 + 4 = 10 224 square metres, just larger than a hectare.

25.1 I subtracted first 10 classes of 28 (280), leaving 364, then another 10, leaving 84, then 2 classes of 28 (56), leaving 28, which was 1 more class. In total this gave 10 + 10 + 2 + 1 = 23 classes.

25.2 I subtracted first 20 coach-loads of 42 (840), leaving 710, then another 10 (420), leaving 290, then 5 coach-loads of 42 (210), leaving 80, which was 1 more coach-load with 38 remainder. In total this gave 20 + 10 + 5 + 1 = 36 coach-loads, with 38 remainder. So, 37 coaches are needed.

26.1 $1 \times 2 \times 3 \times 4 \times 5 = 120 \rightarrow 0.1 \times 0.2 \times 0.3 \times 0.4 \times 0.5 = 0.00120$ (with five figures after the point). This can be written as 0.0012, but don't drop the final zero until *after* you have decided where to put the point.

26.2 $0.6 \times 0.12 \rightarrow 6 \times 12 = 72 \rightarrow 0.072$.

26.3 12% on FSM, 60% of these, no adult in employment.
60% of 12% = 7.2% (50% of 12 is 6, 10% of 12 is 1.2, add these to get 60% of 12).

26.4 $0.71 \times 0.71 = 0.5041$, so the area of the paper is reduced by a factor of about 0.5. This is consistent with the fact that a sheet of A5 paper is half of a sheet of A4.

27.1 $0.126 \div 0.09 = 126 \div 90 = 14 \div 10 = 1.4$ or 140%.

27.2 a) is the same (both numbers doubled), c) is the same (both multiplied by 100), and d) is the same (both divided by 10).

27.3 a) 2 b) 0.2 c) 200 d) 0.002.

28.1 1189 mm = 118.9 cm = 1.189 m.

28.2 a) about 17.5 cm, b) about 20 m, c) about 20 mm.

28.3 70 mph is just over 110 kilometres per hour, so a journey of about 10 000 km will take around 90 hours. Since you also travel through 90° of latitude on this journey, this means that when you are travelling due south (or north) at 70 mph you are moving through about 1° of latitude per hour.

29.1 A5 paper is approximately 149 mm by 210 mm, or 0.149 m by 0.210 m. These dimensions give the area as approximately $0.149 \times 0.21 = 0.03129$ m^2. The fraction $\frac{1}{32}$ as a decimal is 0.03125.

29.2 The volume is 0.175   0.095   0.065 = 0.0010806 m^3, which is just over 0.001 m^3, or one thousandth of a cubic metre, or 1000 cm^3. (NB: 1 litre = 1000 cm^3.)

29.3 1 ha = about $2\frac{1}{2}$ acres. 0.4 ha = 4000 m^2, which could be 40 m   100 m, or 80 m   50 m, 160 m   25 m, and so on.

30.1 £6.50 for half a litre is £13 per litre. £5 for 400 ml is £1.25 for 100 ml, so £12.50 for 1 litre. On this basis, the second is the better buy.

30.2 1.25 dl = 0.125 litres = 125 ml = 25 medicine-spoonfuls.

30.3 a) a quarter of a pound, b) 330 ml, c) 40 litres, d) 70 kg (which is about 11 stone).

30.4 An A4 sheet is $\frac{1}{16}$ of a square metre in area (half of A3, which is half of A2, which is half of A1, which is half of A0, which is 1m^2 in area). So its weight is $\frac{1}{16}$ of 80 g = 5g. A ream is 500 sheets, so weighs 2500 g = 2.5 kg. You can safely put 8 sheets of standard A4 paper (40 g) in an envelope (less than 20 g) and stay within the 60-g limit. This is a useful piece of knowledge!

31.1 The difference is £24,500. The ratio is 42 000:17 500, which could be simplified in various ways, such as 420:175 = 84:35 = 12:5 = 24:10. This is 2.40:1, or £2.40 for every £1.

31.2 The difference is now £25,382. But the ratio is still 2.40:1. Notice that applying a percentage-increase increases the difference, but leaves the ratio the same. Those on higher salaries would prefer this kind of pay rise.

31.3 The difference is still £25,382. But the ratio is now 2.36:1. Notice that applying a flat-rate increase of this kind leaves the difference the same, but reduces the ratio. Those on lower salaries would prefer this kind of pay rise.

32.1 The £4800 must be divided by 8 (1 + 2 + 5), giving £600. The allocations are £600, £1200 and £3000 for nursery, infant and junior respectively.

32.2 Girls: $\frac{4}{7}$ = 57.1% approximately. Boys: $\frac{3}{7}$ = 42.9% approximately. Number of girls = 41 000 ÷ 7   4 = about 23 400. Number of boys = 41 000 ÷ 7   3 = about 17 600.

33.1 61%, 58.8%.

33.2 6543   1.21 and 6543   0.79.

33.3 It makes no difference! The price in both cases is £815.83.
VAT first is: 789   1.175   0.88. Reduction first is: 789   0.88   1.175.

34.1 The increase is £150, which is 12% of £1250.
After decreasing £1400 by 12% we should expect the answer to be less than £1250, because the reduction is 12% of £1400, whereas the previous increase was 12% of only £1250.
Calculator result: £1400   0.88 = £1232, which, as predicted, is less than £1250.

34.2 Science increases by 0.2; 0.2 ÷ 31.6 = 0.006 (approximately) = 0.6%.
Mathematics decreases by 1.8; 1.8 ÷ 32.9 = 0.055 (approximately) = 5.5%.

34.3 The proportion increases by 5 percentage points (54.4 – 49.4). Expressed as a percentage of the starting value, 5 ÷ 49.4 = 0.101 (approximately) = a 10.1% increase over the five years.
The biggest annual percentage increase was 3.7% from 1998 to 1999.

35.1 Note that the increase is not 26 *percentage points*, but 26% of the previous proportion. So 126% (of the previous proportion) = 63 (i.e. 63% of the pupils), which gives 2% = 1, so 100% = 50. So the previous proportion was 50%.

35.2 85.8% = 37 900, so 1% = 37 900 ÷ 85.8, and 100% = 37 900 ÷ 85.8   100.
This gives 44 172.494, which is 44 200 to the nearest hundred.

36.1 The mean class-size for School X is 28.6; the mean class-size for School Y is 27.1. These might be calculated to compare the schools with some national data about mean class-sizes. Or, to see whether, on the whole, the schools are achieving some target for reduction of class-size; but the mean of 28.6 for School X will be little consolation for the teacher with a class of 35! All other things being equal (which is highly unlikely), a comparison might be made between the mean class-sizes of the schools as part of an evaluation of the impact of class-size on pupil achievement.

36.2 The total number of pupils is 129. To find the total number of points, note that there are sixteen 21s, thirty-two 27s, and so on. So, the total is found by calculating (16   21) + (32   27) + (40   33) + (25   39) + (8   45) + (6   51) + (2   57) = 4275. So the mean is 4275 ÷ 129 = 33.1 to one decimal place. Given all the other variables involved, this is a fairly meaningless statistic. It assumes, for example, that a school should get the same credit for two pupils gaining levels 4 and 8 respectively as for two pupils both gaining level 6. There is no valid basis for such an assumption.

37.1 The mode is level 4.

37.2 The mode was level 5.

37.3 The modal interval is £4.00–£5.99.

38.1 This school is in the 'FSM more than 50%' group. Compared to schools in this group, the results in terms of A*–C grades for GCSE mathematics are 'better than average', because their 28% is higher than the median of 18%; i.e. this school did better than at least half of the schools in this group.

38.2 St Anne's devotes 22.1% of the Y3 teaching week to English. This is less than the median percentage for all primary schools. More than half of all schools devote a larger percentage of the Y3 teaching week to English than does St Anne's.

39.1 Compared to schools in this FSM group, with 28% achieving grade C or above in mathematics, the first school has a proportion higher than the UQ (25%). With 15% achieving this level, the second school has a proportion lying between the LQ (12%) and the median (18%). Loosely speaking, the first school has done well compared to similar schools, being in the top quarter based on the proportions of pupils gaining grade C or above in mathematics. The second school is only 'fairly average', below the median but not in the bottom quarter of these schools.

39.2 The percentage of the Y3 teaching week for English at St Anne's (22.1%) is just less than the LQ (22.2%) for all primary schools. This puts them in the bottom quarter of schools in terms of the proportion of the Y3 teaching week devoted to English. The proportion for St Michael's (30%) exceeds the UQ (28.1%) for all primary schools, putting them in the top quarter for this variable. Loosely speaking, St Anne's has a low proportion of the Y3 week devoted to English, whereas St Michael's has a high proportion.

40.1 The 'average' numbers of hours for RE and PE given by the median values are very similar. However, there is much more variation between schools in the time given to PE than there is in the time given to RE. The range for PE (1.7 hours) is greater than that for RE (1.3 hours). Also, the IQR for PE (0.7 hours) is greater than that for RE (0.4 hours), suggesting that the greater variability is not just due to a few schools giving an exceptionally high number of hours to PE.

41.1 A is valid. The maximum marks achieved were 91 and 100 for literacy and numeracy respectively.

B is invalid. The median marks were about 40 and 60 for literacy and numeracy respectively.

C is invalid. The diagram does not tell us anything about how individual pupils did in the tests.

D is invalid. A mark of 40 in literacy was bettered by about 75% of the pupils, whereas 40 in numeracy was the median mark.

E is valid. A mark of 70 in literacy is in the 'fairly average' box, whereas a mark of 70 in numeracy is in the 'high-scoring' whisker.

F is probably valid. The numeracy box (the middle 50% of pupils) and the bottom whisker (the bottom 25%) are substantially lower than the literacy box and lower whisker.

G is invalid. It should be the lowest 50 scores, i.e. the bottom 25%.

H is valid. The top whisker for numeracy is much longer than that for literacy.

42.1 £87.50.

42.2 a) This is not direct proportion: a letter of 40 g will not cost twice as much as one of 20g, for example.

b) This is not direct proportion. A square of side 5 m has an area of 25 m^2; a square of side 10 m has an area of 100 m^2; the ratios 5:25 and 10:100 are not equal.

c) This is direct proportion. Weights in pounds to weights in kg are always in the same ratio, about 1:0.454.

d) This is not direct proportion, although it is sometimes difficult making parents understand this. A pupil at level 2 in Year 2 is unlikely to be at level 4 in Year 4, level 6 in Year 6 and level 8 in Year 8!

42.3 (0,0), (254,100), (127,50)…

The ratio is about 1:0.39 (using a calculator), so the gradient will be about 0.39. This means that 1 cm is about 0.39 of an inch.

43.1 Only (b) might be sensible, but this is assuming that each pupil can be located in one and only one ethnic origin group and that there are not too many different groups.

A pie chart could not be used for (a) because there is not a single population to be represented by the pie. A pie chart would be hopeless for (c) because there are 52 subsets!

43.2 a) third b) half c) walking.
d) bus (because 48 pupils = 20%).

44.1 a) About 61.
b) About 28.

The transposition of the digits here makes these two results easy to remember as reference points for converting temperatures.

In (a) the 9C/5 must be calculated before adding on 32.

In (b) the (F – 32) bracket must be done first, before multiplying by 5 and dividing by 9.

45.1 The first purpose of the graph is to compare the total numbers of pupils per A-level subject, shown by the heights of the columns. The second purpose is to show the contributions of boys and girls to these totals. For example, music is clearly the lowest frequency of these five subjects, but within this the contribution of the girls can be seen to be far greater than that of the boys.